On the influence of one-point-bend impact test parameters on dynamic stress intensity factor variation

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Abstract

Mathematical model of the specimen deformation during one-point-bend impact test is proposed. Using mode superposition technique, a closed form solution for dynamic stress intensity factor (DSIF) variation with time is derived. Parameters used in this formula are determined for plane stress finite element model of the specimen. Theoretical DSIF-time curve is compared with experimental one registered by Böhme and Kalthoff using caustics method. Influence on DSIF of the specimen geometry parameters and specimen-striker contact stiffness is investigated. It is shown that increasing of contact stiffness causes increasing DSIF growth rate. Increasing crack relative depth and/or specimen length leads to higher maximum DSIF values.

Keywords: Dynamic Fracture Toughness, Dynamic Stress Intensity Factor, Instrumented Impact Test, One-Point-Bending

1 Introduction

Instrumented impact fracture testing of precracked beam specimens is widely used for determination of dynamic fracture toughness of brittle materials. The aim of the test is to determine the critical value of the dynamic stress intensity factor (DSIF) $K_{ld}$, i.e., to obtain DSIF-time relation $K_I(t)$ and to register the moment of crack initiation. There are two main differences between static and dynamic fracture tests. First, for the early stage of the specimen dynamic deformation DSIF (contrary to quasi-static SIF) is not proportional to the acting force. Thus, to evaluate $K_I(t)$ precisely, full scale dynamic analysis has to be carried out at least on the initial stage of the specimen deformation when transient dynamic effects are the most noticeable. Experimental results [1] and numerical analysis [2] have shown, that during this stage of loading the specimen may lose contact with the supports. At this time bending of the specimen is not three-point but one-point one, i.e., it is caused by tup loading and specimen inertia. That is the reason why the type of impact tests in which crack growth is initiated during this initial period of time$^1$, has been called one-point-bend test [1, 3, 4].

Second, contrary to static case, where the specimen geometry is very precisely described in corresponding standards, in dynamic tests wide ranges of the specimen relative lengths and crack depths were used. Differences in specimen geometry lead to more or less desirable differences in $K_I(t)$. The aim of this work was to derive the mathematical model of the specimen

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$^1$This is the case for high velocity testing of brittle plastics, ceramics or hardened steels.
deformation during one-point-bend test and to investigate the influence of the specimen geometry and the specimen-striker contact stiffness on $K_I(t)$.

2 Theory

The following assumptions were used to model the specimen interaction with striker:

- specimen material is linearly elastic;
- specimen-striker interaction is modeled by point force $F(t)$;
- striker is perfectly stiff and has constant curvature of the tip;
- contact between the specimen and striker is described by Hertz’s law.

Let’s consider the specimen with length $L$, width $W$, thickness $B$ and crack length $a$ (Fig ??). Utilizing classic Timoshenko’s approach, one may assume that the force experienced by the striker of drop weight or pendulum machine is equal to Hertzian contact force, so

$$F(t) = k\alpha^{3/2}$$

where $k$ is contact stiffness and $\alpha$ is the depth of penetration of the striker into the specimen surface. When the impact velocity is small and specimen is sufficiently stiff, $\alpha$ is usually not too large to make the linearization of Eqn.(1) implausible. Thus, if $k^*$ is the linearized contact stiffness, we may use

$$F(t) = k^*\alpha$$

instead of Eqn.(1).

The depth of specimen penetration can be defined as

$$\alpha = \begin{cases} u_{str}(t) - u_F(t) & u_{str}(t) \geq u_F(t) \\ 0 & u_{str}(t) < u_F(t) \end{cases}$$

where $u_{str}(t)$ is the striker displacement, $u_F(t)$ is the $x$-component of the specimen displacement in the contact point. The following equation can be obtain easily for the striker position

$$u_{str}(t) = v_0t - \frac{1}{M} \int_0^t F(\tau)(t - \tau) \, d\tau$$

where $v_0$ is the impact velocity, $M$ is the mass of the striker. In Eqn. (4) the effect of increasing the striker velocity during impact due to gravitation (that is the case for drop weight machine) is neglected.
If the eigenfrequencies $\omega_i$ and normalized nontrivial symmetrical eigenmodes $\psi_i(x, y)$ are known for the unsupported specimen, its $x$-component of displacement vector in the contact point can be written as

$$u_F(t) = \frac{1}{m} \int_0^t F(\tau)(t - \tau) d\tau + \sum_{i=1}^{\infty} \frac{(\psi_F)_i^2}{\omega_i} \int_0^t F(\tau) \sin(\omega_i(t - \tau)) d\tau$$  \hspace{1cm} (5)

where $(\psi_F)_i$ is the $x$-component of the $i$th symmetrical specimen eigenmode in the contact point.

Combining Eqs.(2) through (5) we arrive at:

$$\frac{F(t)}{k^*} = v_0 t - \frac{1}{m} \int_0^t F(\tau)(t - \tau) d\tau - \sum_{i=1}^{\infty} \frac{(\psi_F)_i^2}{\omega_i} \int_0^t F(\tau) \sin(\omega_i(t - \tau)) d\tau$$  \hspace{1cm} (6)

where $\bar{m} = Mm/(M + m)$. This equation is valid for $u_{str}(t) \geq u_F(t)$, otherwise $F(t) \equiv 0$.

Let us consider a test specimen as a linear dynamic system and interpret $K_1(t)$ as a form of the specimen response to external excitation $F(t)$. Thus, the excitation-response relationship can be expressed by convolution integral

$$K_1(t) = \int_0^t F(\tau) h_K^{(1)}(t - \tau) d\tau$$  \hspace{1cm} (7)

where $h_K^{(1)}(t)$ is the opening mode DSIF-response of the unsupported specimen to the unit impulse from the tup, $F(t) = 0$ for $t \leq 0$. Using mode superposition technique the following formula for $h_K^{(1)}(t)$ has been derived [5, 6]

$$h_K^{(1)}(t) = k_K^{(1)} \sum_{i=1}^{\infty} \eta_i^{(1)} \omega_i \sin \omega_i t$$  \hspace{1cm} (8)

where $k_K^{(1)}$ is SIF for static one-point bending of the specimen by unit uniformly distributed volume force (see Fig 1), $\eta_i^{(1)}$ is the weight coefficient proportional to the $i$th symmetric mode contribution into $k_K^{(1)}$. After substituting (8) into (7) the following relation can be obtained

$$K_1(t) = k_K^{(1)} \sum_{i=1}^{\infty} \eta_i^{(1)} \omega_i \int_0^t F(\tau) \sin(\omega_i(t - \tau)) d\tau$$  \hspace{1cm} (9)

Non-symmetrical eigenmodes are ignored because they do not cause crack opening and non-zero SIF arising...
Equations (6),(9) are the Volterra integral equations of the second kind with convolution type kernel. These equations can be easily solved using Laplace transform technique. Combining transformed Eqs. (6),(9) and denoting $\Omega^2 = k^*/m$ one can obtain the following equation for transform $\mathcal{L}[K_1(t)]$ of $K_1(t)$

$$
\mathcal{L}[K_1(t)] = v_0 k^* k_s^{(1)} \omega^2 \frac{\sum_{i=1}^{\infty} \eta_i^{(1)} \omega_i^2}{\Omega^2 + s^2 \left(1 + \sum_{i=1}^{\infty} \frac{(\psi_i)^2}{s^2 + \omega_i^2}\right)}. \quad (10)
$$

Usually only a few eigenmodes have to be taken into account in Eqs. (6),(9) to obtain the result with reasonable accuracy. If $N_F$ and $N_K$ are the numbers of eigenmodes considered in (6) and in (9) respectively, Eqn. (10) can be rewritten as

$$
\mathcal{L}[K_1(t)] = v_0 k^* k_s^{(1)} \omega^2 \frac{\sum_{i=1}^{N_K} \eta_i^{(1)} \omega_i^2}{\Omega^2 + s^2 \left(1 + \sum_{i=1}^{N_F} \frac{(\psi_i)^2}{s^2 + \omega_i^2}\right)}. \quad (11)
$$

Inverse Laplace transform for this equation can be performed numerically. However, for $N_F = 1$ the following analytical solution have been obtained

$$
K_1(t) = v_0 k^* k_s^{(1)} \left\{ \frac{\omega^2}{4C_1C_2} \left[ \left( \sum_{i=1}^{N_K} \eta_i^{(1)} \left( C_3 - (C_1 - C_2)^2(1 - \omega_i^2/\omega_1^2) \right) \right) \frac{\sin(C_1 - C_2)t}{C_1 - C_2} - \right. \\
- \left. \left( \sum_{i=1}^{N_K} \eta_i^{(1)} \left( C_3 - (C_1 + C_2)^2(1 - \omega_i^2/\omega_1^2) \right) \right) \frac{\sin(C_1 + C_2)t}{C_1 + C_2} \right] + \\
+ \sum_{i=2}^{N_K} \frac{\omega_i \eta_i^{(1)} (1 - \omega_1^2/\omega_i^2) \sin(\omega_i t)}{C_3 + \Omega^2(1 - \omega_i^2/\omega_1^2)} \right\}. \quad (12)
$$

where

$$
C_1 = 1/2 \sqrt{k^* (\psi_1)^2 + (\Omega + \omega_1)^2} \quad C_2 = 1/2 \sqrt{k^* (\psi_2)^2 + (\Omega - \omega_1)^2}
$$

$$
C_3 = k^* (\psi_1)^2 + \omega_1^2 - \omega_i^2
$$

For $N_K = 1$ Eqn. (12) can be reduced to

$$
K_1(t) = \frac{v_0 k^* k_s^{(1)} \omega^2 \eta_1^{(1)}}{4C_1C_2} \left( \frac{\sin(C_1 - C_2)t}{C_1 - C_2} - \frac{\sin(C_1 + C_2)t}{C_1 + C_2} \right). \quad (13)
$$

Of course, both these solutions are valid only when the specimen is in contact with striker.

### 3 Results

To check the accuracy of the formulas derived, theoretical $K_1(t)$ curves have been calculated and compared with experimental data. Böhme and Kalthoff [1] reported results of one-point-bend test, in which Araldite B specimen with $L = 412$ mm, $W = 100$ mm, $B = 10$ mm and
\[ K_I(t), \text{ MPa m}^{1/2} \]

\[ a = 30 \text{ mm was loaded by falling tup with } M = 4.9 \text{ kg and radius of the striker tip curvature 8 mm. DSIF was measured using caustics method. Due to the low impact velocity } (v_0 = 1 \text{ m/s}) \text{ and using blunted notch instead of crack, crack initiation was not observed in this test.} \]

Numerical $K_I(t)$ for this test has been calculated from Eqn. (12). The following procedures have been used to obtain the values of parameters used in this formula:

- Linearized stiffness of the striker $k_1^*$ has been estimated as 6.6 MN/m using the method proposed in [7].

- Values of $k_s^{(1)}, \omega_i$ and $\eta_i$ were calculated using simple formulas obtained previously by fitting the results of finite element analysis (FEA) for 2D model of the specimen [6].

- Values of $\psi_F^i$ were obtained from direct FEA using ADINA 6.1.

Experimental and numerical DSIF curves are compared in Fig. 2. It was found that using $N_F > 1$ and $N_K > 3$ has negligible influence on improving the accuracy of numerical $K_I(t)$ for the test considered. The simplest one-mode approach (line 3) (i.e., using formula(13)) leads to underestimating $K_I(t)$ for the initial stage of its growth. Results for $N_K = 3$ calculated for the same $k_1^*$ (line 4) are in better agreement with the experimental curve.

The influence of $k_1^*$ variation on $K_I(t)$ has been investigated for the test considered. Increasing the striker stiffness leads to increasing DSIF growth rate (compare lines 2,4,5). It is interesting to note that the maximum DSIF values for $k_1^* = 5 \ldots 10 \text{ MN/m are nearly the same (the differences are smaller than 5 per cent).}$

For $k_1^* = 2 \text{ MN/m an influence of increasing the specimen length to } L = 550 \text{ mm and/or increasing the crack depth to } a = 50 \text{ mm on } K_I(t) \text{ has been investigated}^3. \text{ Deeper crack causes increase the maximum DSIF value which can be reached during the test (see Fig. 3). Increasing relative specimen length also leads to higher maximum DSIF but decreases DSIF growth rate.}

\[ ^3 \text{Such a low value of striker stiffness has been used to except loosing the contact between the striker and more flexible specimen} \]
4 Conclusions

Mathematical model of the specimen deformation during one-point-bend impact test is proposed. Using mode superposition technique two integral equations for specimen/striker contact force and DSIF determination are derived. For one-mode estimation of the contact force, a closed form solution for DSIF determination is derived. Theoretical DSIF-time curve satisfactorily agree with experimental one measured by Böhme and Kalthoff using caustics method for large scale Araldite B specimen.

Influence of the specimen geometry and specimen-striker contact stiffness has been investigated. It is shown that increasing of contact stiffness causes increasing DSIF growth rate. However, the maximum DSIF value does not change significantly for wide range of contact stiffness values. Increasing crack relative depth leads to higher maximum DSIF value. The same effect has increasing specimen length but in this case DSIF growth rate decrease.

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References


